Analysis of the trade-off between resolution and bandwidth for a nanoforce sensor based on diamagnetic springs.

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Only micro-nano force effects can be directly measured

A force transducer is needed

Deformation
of an elastic µstructure
when a force is applied on it

Displacement
of a rigid seismic mass
when a force is applied on it

Displacement measured with appropriate sensors

Knowing the displacement, the force must be reconstructed
Sensor configuration

- **Force measured:** along x axis
- **Stiffness:** 0.005 N/m to 0.03 N/m
- **Typ. resolution:** 1 to 5 nN
- **Range:** 1 nN to 40 μN
- **Mass:** 20 to 80 mg
- **Typ. resonant frequency:** 3 Hz

Maglevtube (seismic mass)
Passive force sensor

\[ F(t) \rightarrow \text{Transducer} \rightarrow x(t) \rightarrow \text{Deconvolution UIO} \rightarrow \hat{F}(t) \]

Force to determine: \( F(t) \)

Displacement measured: \( x(t) \)

A priori information: \( \)  

Environmental noise: \( \)  

Measurement noise: \( \)  

Estimated force: \( \hat{F}(t) \)
Environmental noise

\( F_x(t) \)

Transducer

\( x(t) \)

STIL confocal chromatic sensor

Measurement noise \( v_k \)

\( m_k^x \)

Kalman filter

\( \hat{F}_k \)

Confocal chromatic sensor (CL2 + MG140)

Zero-mean white gaussian noise \( v_k \)

Variance: \( E[v_k^2] = R \)

Typ. \( R = 1.44 \times 10^{-16} \text{ m}^2 \)

- Discretized uncertainty model for \( F_x(t) \)
- Discretized transducer model (2nd order dynamic)
- Measurement noise model
Time-varying Kalman filter synthesis:


Deconvolution of a noisy output: introduce a necessary trade-off between resolution and bandwidth

Driven here by a single scalar parameter \( (N^2/\text{Hz}) \): \( \tilde{W}_F \)

\[
\phi_{\omega,\omega}(\tau) = \tilde{W}_F \delta(\tau) \quad \forall \tau \in \mathbb{R}
\]

Power Spectral Density (PSD) chosen by the end-user

Uncertainty modeling of the input force

\( \hat{F}(t) = \omega(t) \)
Force estimation

Input force model 1 (uncertain)

\( W_{\dot{F}} \)

\( \omega(t) \)

\( \int \)

\( F(t) \)

\( B \)

\( + \)

\( \dot{X}(t) \)

\( \int \)

\( X(t) \)

\( C \)

\( x(t) \)

Transducer model 2 (deterministic)

\( \dot{A} \)

\( A \)

\( \dot{x}(t) \)

\( x(t) \)

Extended state-space model 3 including the uncertain modeling of the force

\[ X^e(t) = \begin{bmatrix} x & \dot{x} & F \end{bmatrix}^T \]

Discretization

\( X^e_k = \begin{bmatrix} x_k & \dot{x}_k & F_k \end{bmatrix}^T \)

Measurement noise variance:

\( R \)

Ext. state uncertainty cov. matrix:

\( Q \)

(matematical consequence of 1 + 2 merging)

\( Q = W_{\dot{F}} \eta(T_s) \)

(x(t) measurement:

\( m^x_k \)

Time-varying Kalman filter

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Study for fixed values of $W_{\tilde{f}}$ and $T_s$ (and independence of *a priori* knowledge on $X_0^e$)

A 3rd order-state equation:

$$
\hat{X}_{k+1|k} = A^K \hat{X}_{k|k-1} + B^K m^x_k
$$

$$
\hat{F}_k = C^K \hat{X}_{k|k-1} + D^K m^x_k
$$

$A^K$, $B^K$, $C^K$, $D^K$ are functions of $R$ and $Q = W_{\tilde{f}} \eta(T_s)$
Corresponds to the level of noise $n_k$ in the force estimation

\[ x_k = 0 \]
\[ m_k^x \]

Transducer displacement

Measurement noise

Steady-state Kalman filter

\[ \hat{F}_k \]
\[ n_k \]

\[ \nu_k \]

$\nu_k$ substituted to $m_k^x$ in previous state-equation

\[ \hat{X}_{k+1|k} = A^K \hat{X}_{k|k-1} + B^K \nu_k \]
\[ n_k = C^K \hat{X}_{k|k-1} + D^K \nu_k \]

$n_k$ dynamic

$n_k$ statistical properties

Mean

\[ \mu_k = 0 \quad \forall k \]

Variance

\[ \Sigma_k = C^K S_k C^K^T + D^K R D^K^T \]
\[ S_{k+1} = A^K S_k A^K^T + B^K R B^K^T \]
Force sensor resolution study

$f_s = 1000$ Hz

$n_k$ standard deviation versus $W_F$

$W_F$

$\sigma_{k0.5} (N)$

Power spectral density (N$^2$/Hz)

$10^{-18}$ $10^{-17}$ $10^{-16}$ $10^{-15}$

Force estimation (N)

estimated force

mean

99% confidence interval

$W_F = 10^{-19}$

$W_F = 10^{-17}$

time (sec.)

0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8

-2.54 $\times$ 10$^{-7}$

-2.55

-2.56

time (s)

0 1 2 3 4 5

-2.54 $\times$ 10$^{-7}$

-2.55

-2.56
Force sensor bandwidth study

\[ F^x(t) \rightarrow \text{ADC} \rightarrow \text{Identified discretized Transducer} \rightarrow x_k \rightarrow m_k \rightarrow \text{Steady-state Kalman filter} \rightarrow \hat{F}_k \]

Measurement noise \( v_k \)

\[ x_k = \begin{bmatrix} X_k \\ \hat{X}^e_k |_{k-1} \end{bmatrix} \]

\[ x_{k+1} = A_g x_k + B_g \begin{bmatrix} F^x_k \\ v_k \end{bmatrix} \]

\[ \hat{F}_k = C_g x_k + D_g \begin{bmatrix} F^x_k \\ v_k \end{bmatrix} \]

Associated transfer function

\[ \frac{\hat{F}(e^{j\omega})}{F^x(e^{j\omega})} \]

with \( v_k = 0 \)
Force sensor bandwidth study

$f_s = 1000$ Hz

- $W_F = 10^{-18}$
- $W_F = 10^{-17}$
- $W_F = 10^{-16}$
- $W_F = 10^{-15}$

Measurement noise $v_k = 0$

Transducer displacement

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Transducer resonant frequency: 3 Hz
The force estimation has to take into account the behavior due to the mass inertia

Estimation processing driven by one parameter

The parameter effect on the trade-off resolution / bandwidth is fully characterized

Design Drawbacks

Open-loop design

Extreme sensitivity to external disturbing forces (seismic and subsonic vibrations, …)

In progress

New modeling including these disturbances

Future design with real-time disturbances measurement and closed-loop disturbances compensation


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