# Controlled Optical Trapping of Nanoparticles

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# Outline

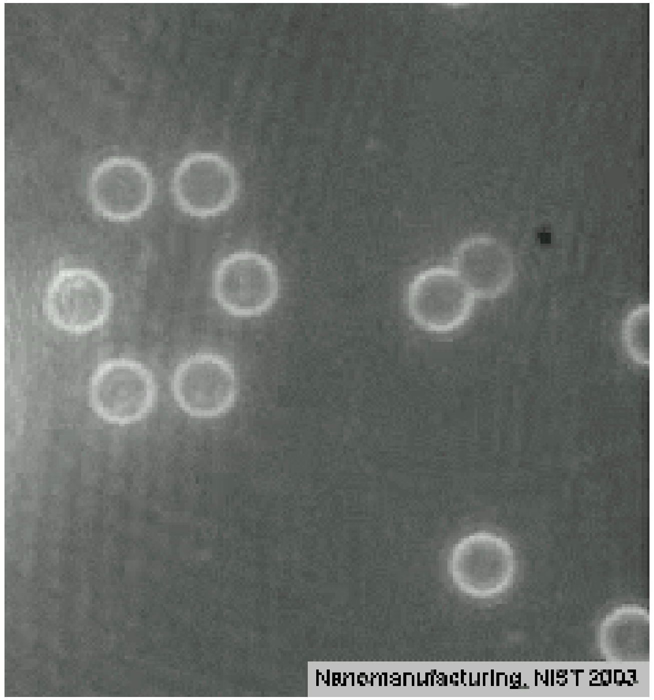
- Optical Trapping
  - Fundamentals
  - Trapping Capabilities
  - Applications
- Controlled Optical Trapping
  - Why Feedback Control?
  - Trapping Instrumentation
  - Dynamic Modeling
  - Scanning Control
- Conclusions and Future Work

# Optical Trapping

Method for manipulating micro- and nanoscale particles with optical forces in solution, vacuum, and air.

- Optical Tweezers
  - Fast and Accurate
  - Cost-effective
  - Bio-compatible
  - Manipulate structures from micrometers to 25 nm or below
  - Apply forces on the order of pN
  - Trap conductors, semiconductors and dielectrics

*Goal:* Provide modeling, systems integration, automation, and control systems to enable the transition from a scientific instrument to a robust nanomanufacturing tool.



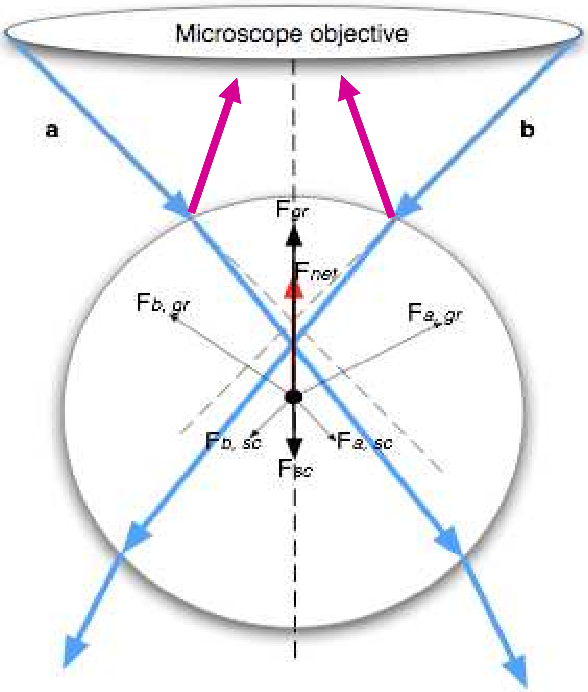

Time-shared optical traps with 3 µm polystyrene particles

# Description of Trapping Forces

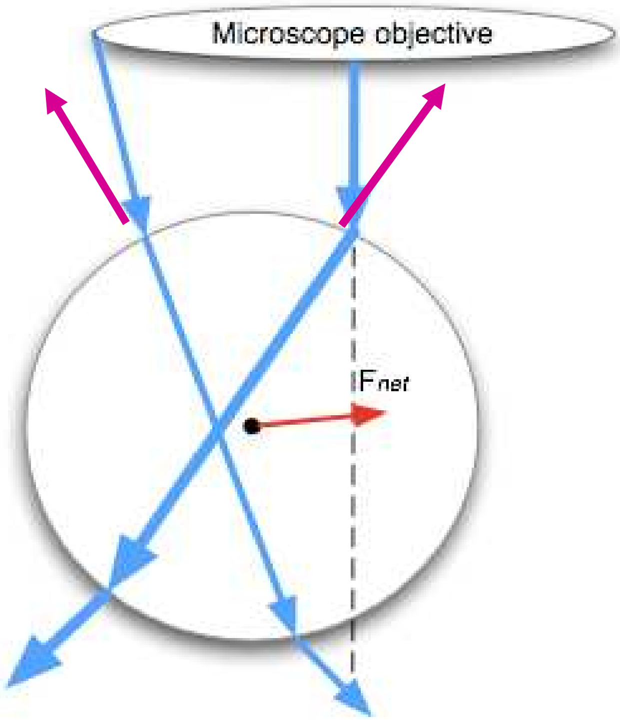
Trapping forces are:

- Axial and transverse
- Composed of scattering and gradient forces
- Trap is stable when gradient forces > scattering forces (need high NA objective)
- Function of particle material and size

**Axial Forces**



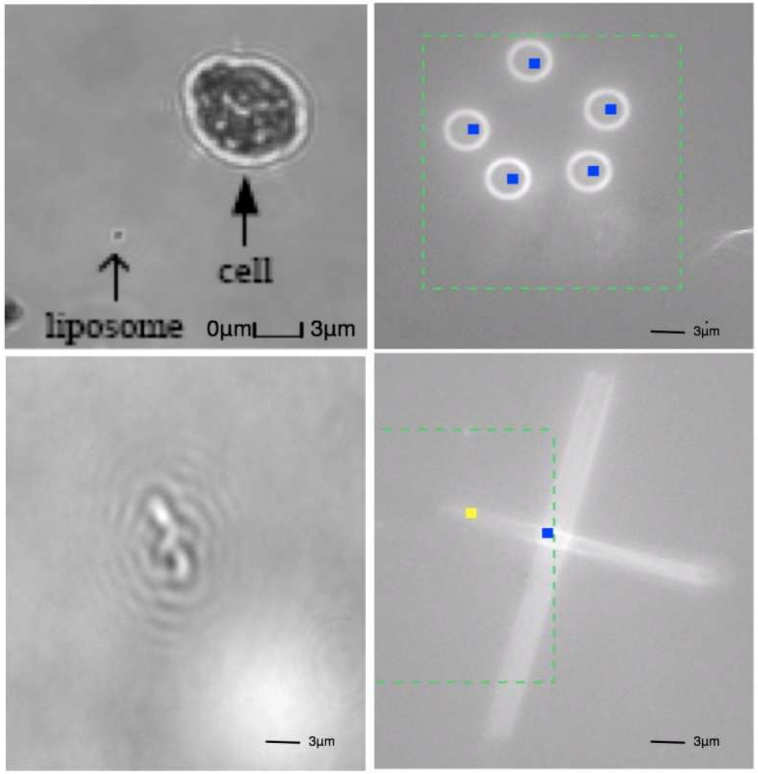

**Transverse Forces**



# Optical Trapping Capabilities

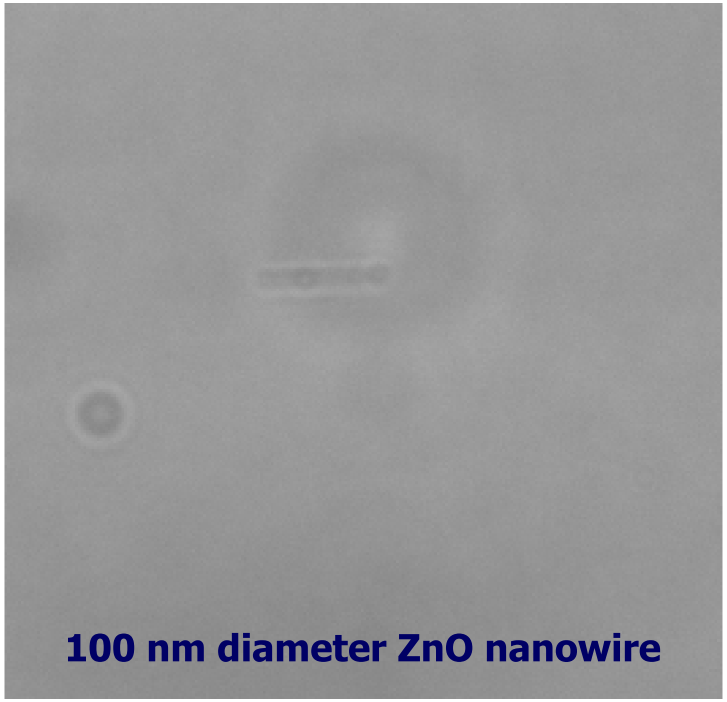
## Multi-Component Trapping

biological particles



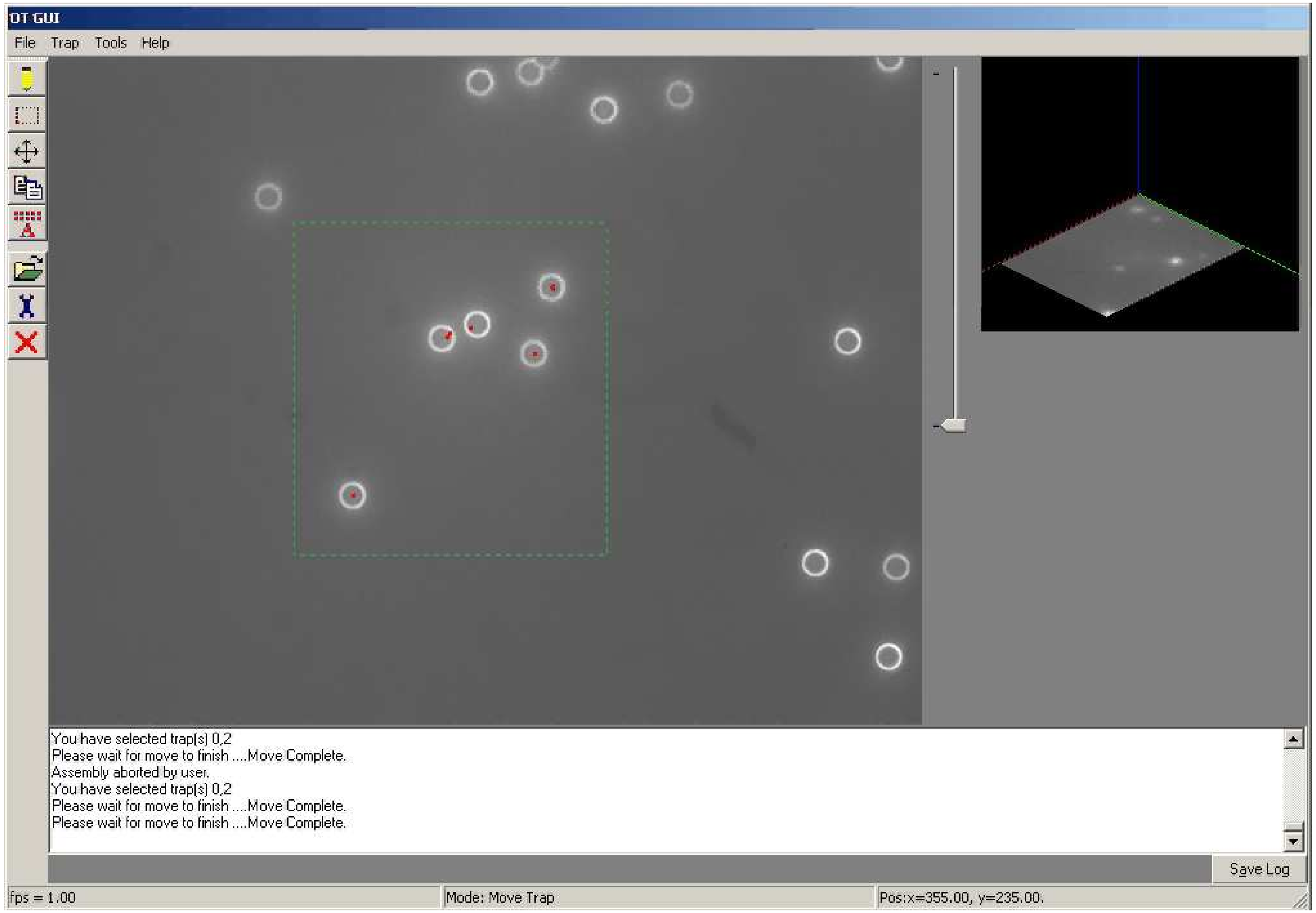

nanowires microcrystals

## Multi-DOF Nanowire Manipulation




*Time-Shared Trapping*: Laser is scanned at high speed to produce multiple point traps and/or line traps. Assuming the laser scans faster than the response of the particle, multiple stable traps are created.

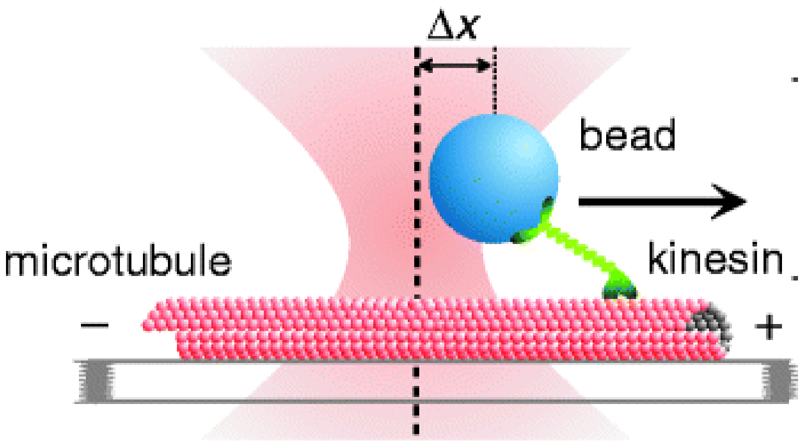
# Automated Assembly



# Biophysical Measurements

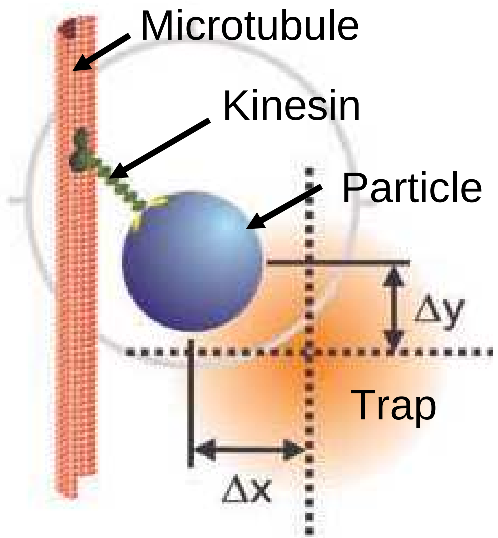
- Optical traps can track the motion of macromolecules and apply pN-level forces to measure their chemomechanical properties.
- Position and force clamps are two approaches that use feedback to perform these measurements

**Position Clamp**
**(constant position, varying force)**




K. Visscher et al., Nature, 1999.

**Force Clamp**
**(constant force, varying position)**




M. Lang et al., Biophysical Journal, 2002.

# Creating Particle Assemblies

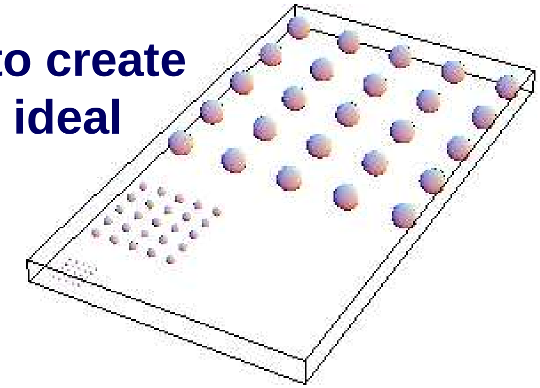
***Traditional approaches to creating particle standards start to fail at dimensions below a few micrometers.***

**For better standards you must control**

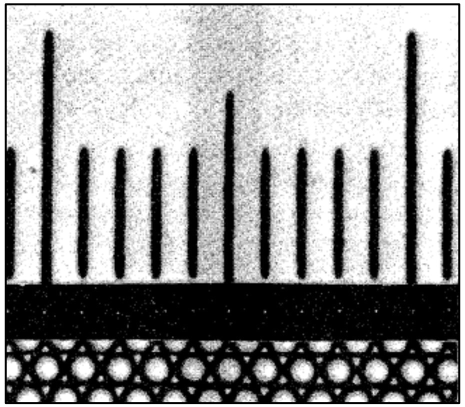
- Surface chemistry
- Particle selection
- Particle placement

**A new way to create more nearly ideal standards**

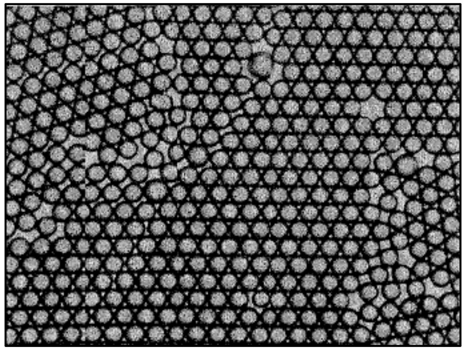
- Dimensional
- Mechanical
- Form



Direct dimensional measurement of microspheres requires nearly-perfect crystalline arrays.

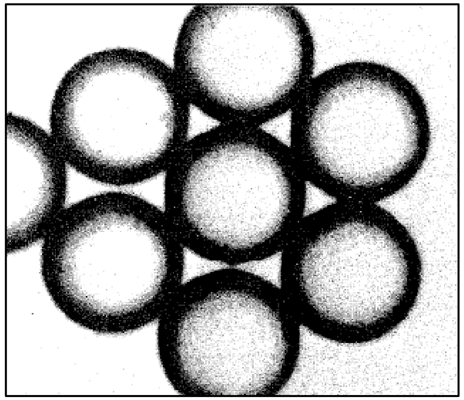


However, self-assembled arrays always have defects

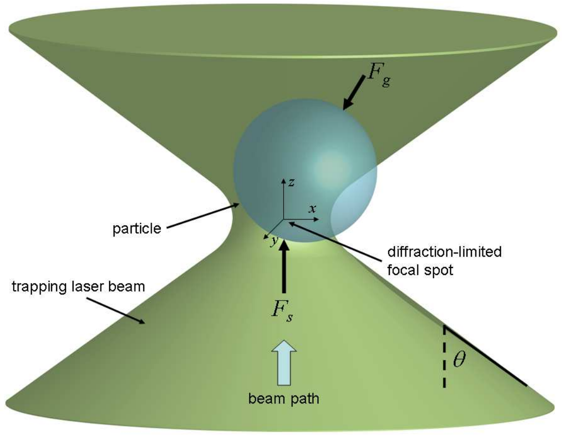


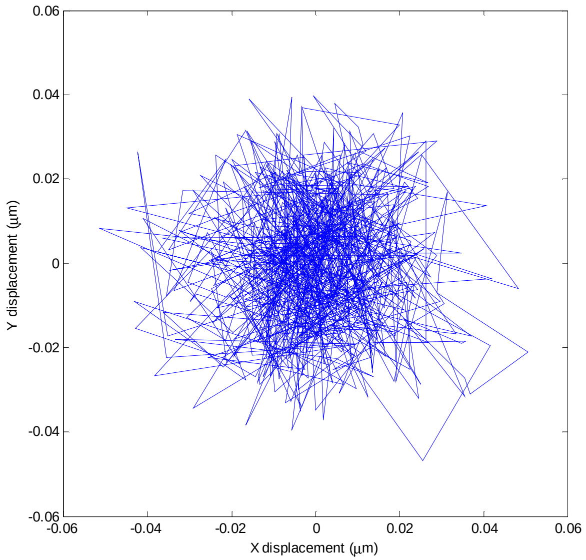
**Uncertainty is Increased by:**

- Air gaps
- Contact flattening
- Swelling
- Surfactant contamination



# Why Feedback Control?




*Brownian Motion*

- Thermal noise causes large deviations of the particle from the center of the trap
- The trap potential depth is finite so these excursions can result in losing the particle

*Limited Trapping Force*

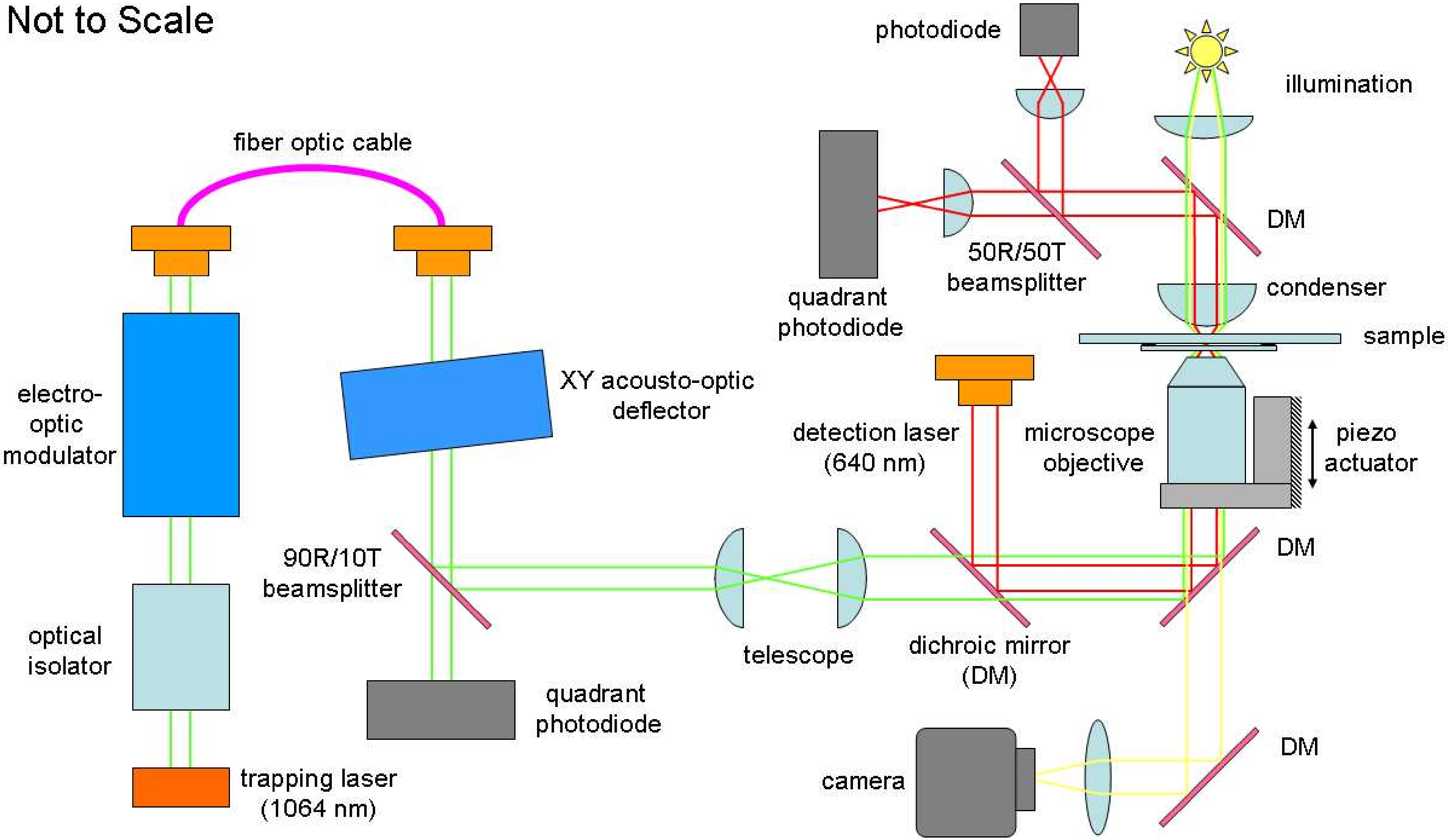
- As the particle size decreases, so does the trapping force
- The result is that the trapping lifetime becomes shorter for smaller and smaller particles
- Currently, particles in the range of 10 – 50 nm can only be trapped for a few seconds
- Feedback is one way to extend the trapping lifetime

*Trajectory Control*

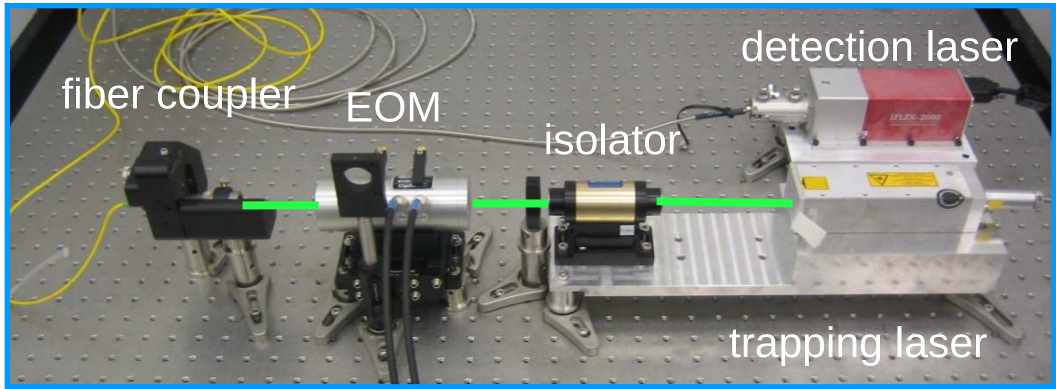
- Feedback can also be used to improve motion tracking at the nanoscale

# Optical Layout

Not to Scale

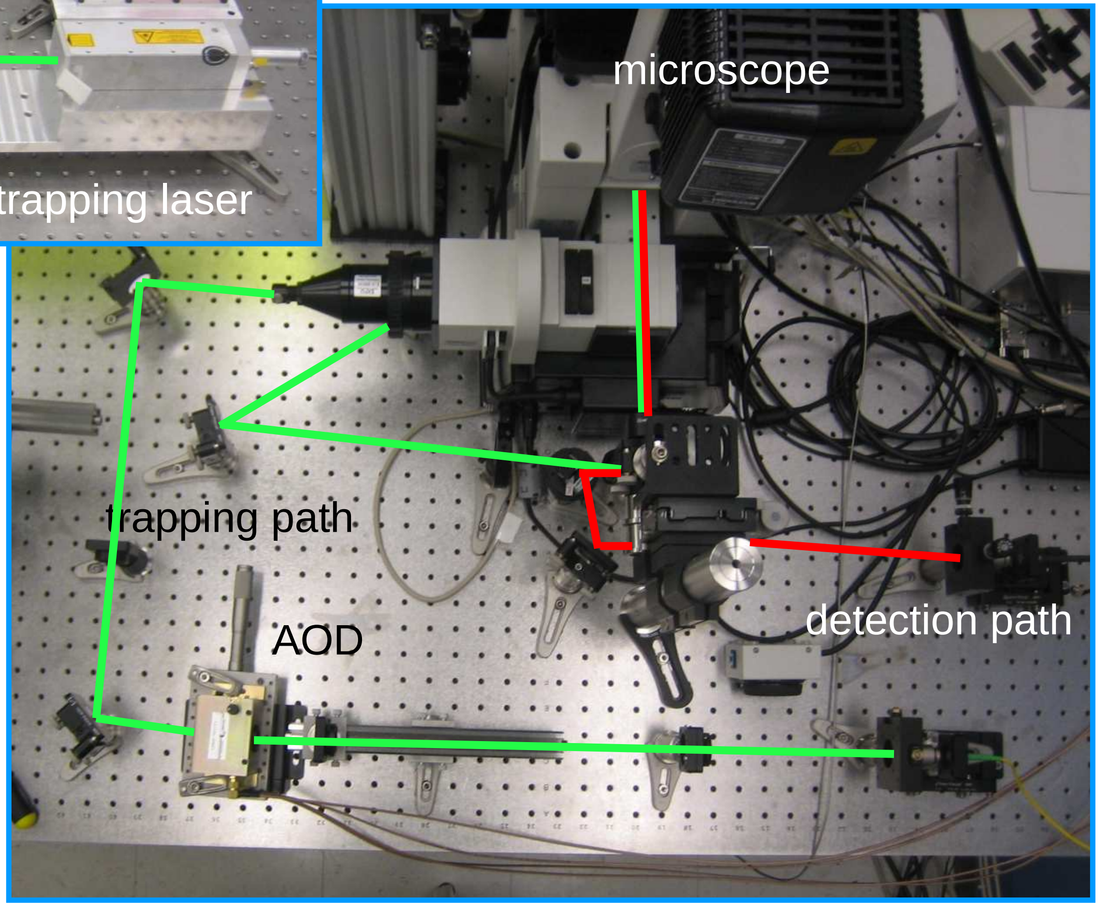


# Optical Layout




Fiber-coupled lasers with EOM

Main instrument table with AOD




Specifications

IR Laser: 3 W / 150 mW at sample

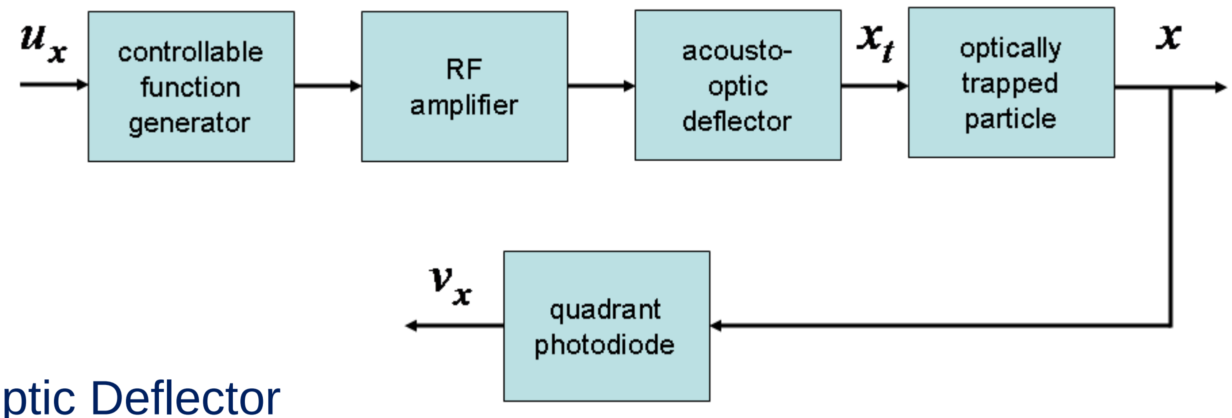
Detection Laser: 1 mW

AOD: 30 mrad range, 30 kHz bandwidth

EOM: 250 kHz bandwidth, 500:1 extinction ratio

QPD: 100 kHz bandwidth

# Open-Loop Block Diagram




Acousto-Optic Deflector

- Has a flat frequency response up to 30 kHz
- However, there is a 100 µs time delay in the AOD and drive electronics
- Model: $x_t(s) = k_{aod_x} e^{-T_{d_x} s} u_x(s)$

Signal Path

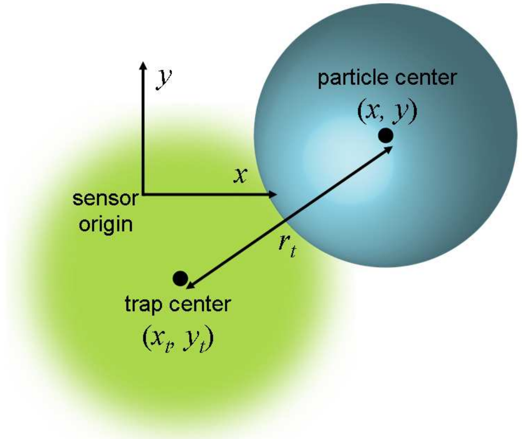
- An applied voltage changes the RF frequency sent to the AOD
- The change in frequency changes the scan angle, moving the trap in the sample
- The displacement of the particle due to the trap motion is measured by the QPD.

# 2D Trap Model

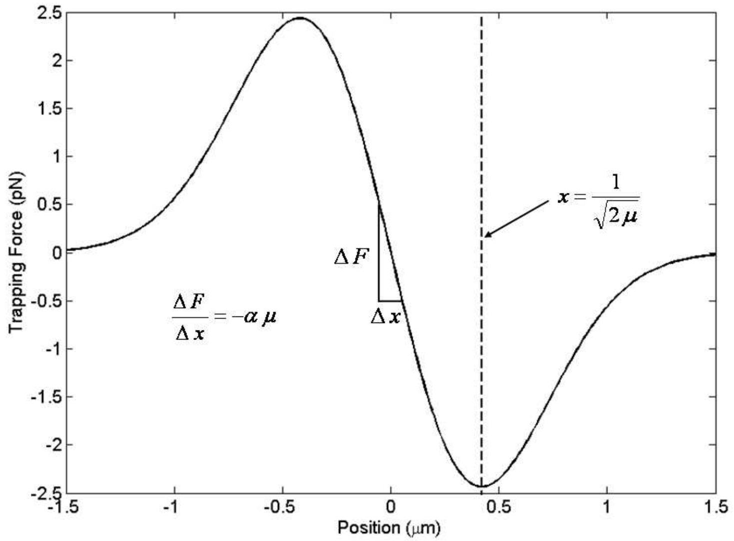
## Trap Characteristics

- Trap stiffness is nonlinear (PE is approximately a Gaussian function)
- Width of Gaussian is a function of the focal spot and particle size
- System is overdamped in a fluid
- Thermal noise is "white"

### Reference Frame




### Trap Stiffness




### Equations of Motion

$$m\ddot{x} + \beta\dot{x} + \alpha\mu\left(x - x_t\right)e^{-\mu r_t^2} = \gamma\Gamma_x(t)$$

$$m\ddot{y} + \beta\dot{y} + \alpha\mu\left(y - y_t\right)e^{-\mu r_t^2} = \gamma\Gamma_y(t)$$

where:

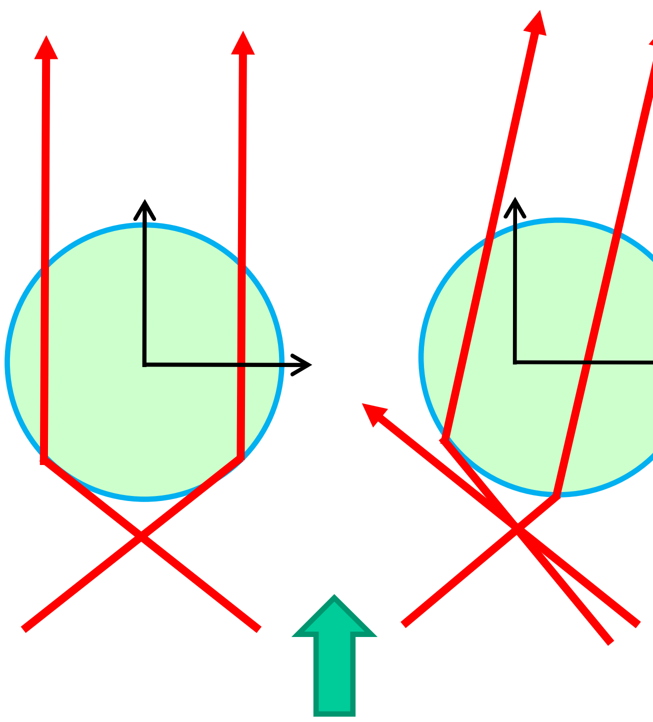
$$\gamma = \left(2\beta k_B T\right)^{1/2} \qquad r_t = \left(\left(x - x_t\right)^2 + \left(y - y_t\right)^2\right)^{1/2}$$

# Detection System

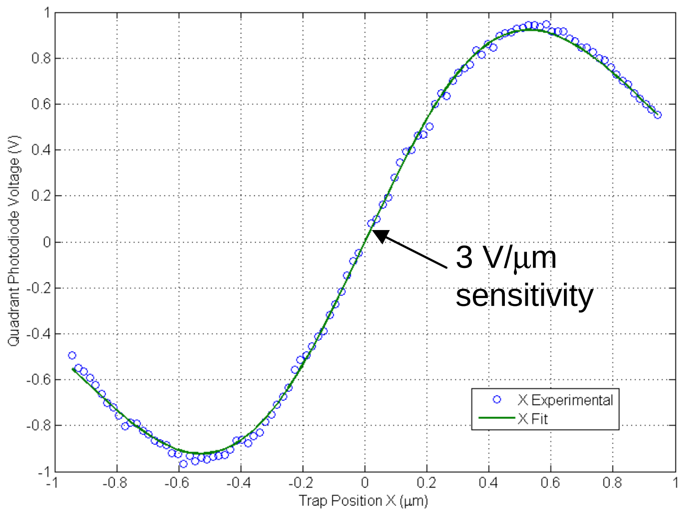
Particle Position Measurement

- Back-focal-plane detection provides nm position resolution feedback
- Light collimated by the particle is imaged onto a quadrant photodiode

Ray Optics Description




Voltage vs. Particle Position




Detector Model

$$v_x = k_x x e^{-\varepsilon_x x^2}$$

# Reduced-Order Model

Assumptions
- Mass of the particle is negligible
- The system is overdamped
- The time delay is negligible
- The trap and detector can be linearized

Trap Dynamics

$$\dot{x} = -\frac{\alpha\mu}{\beta}(x - x_t) + \frac{\gamma}{\beta}\Gamma_x(t)$$

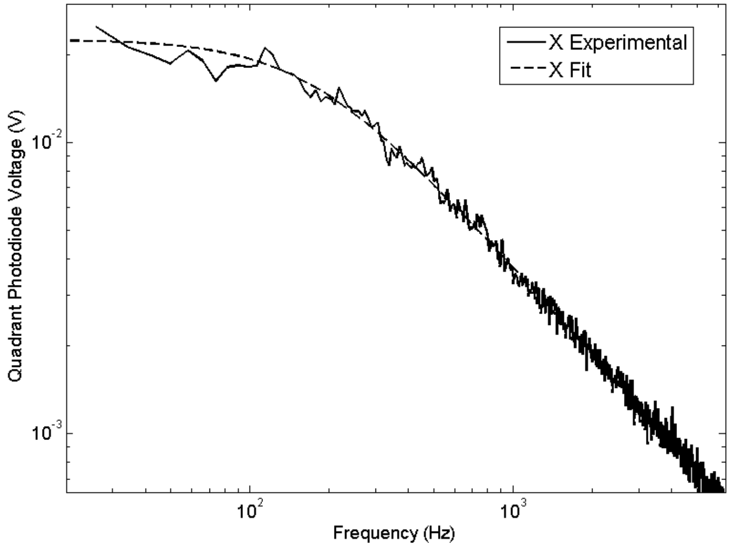
AOD

$$x_t = k_{aod_x} u_x$$

Detector

$$v_x = k_x x$$

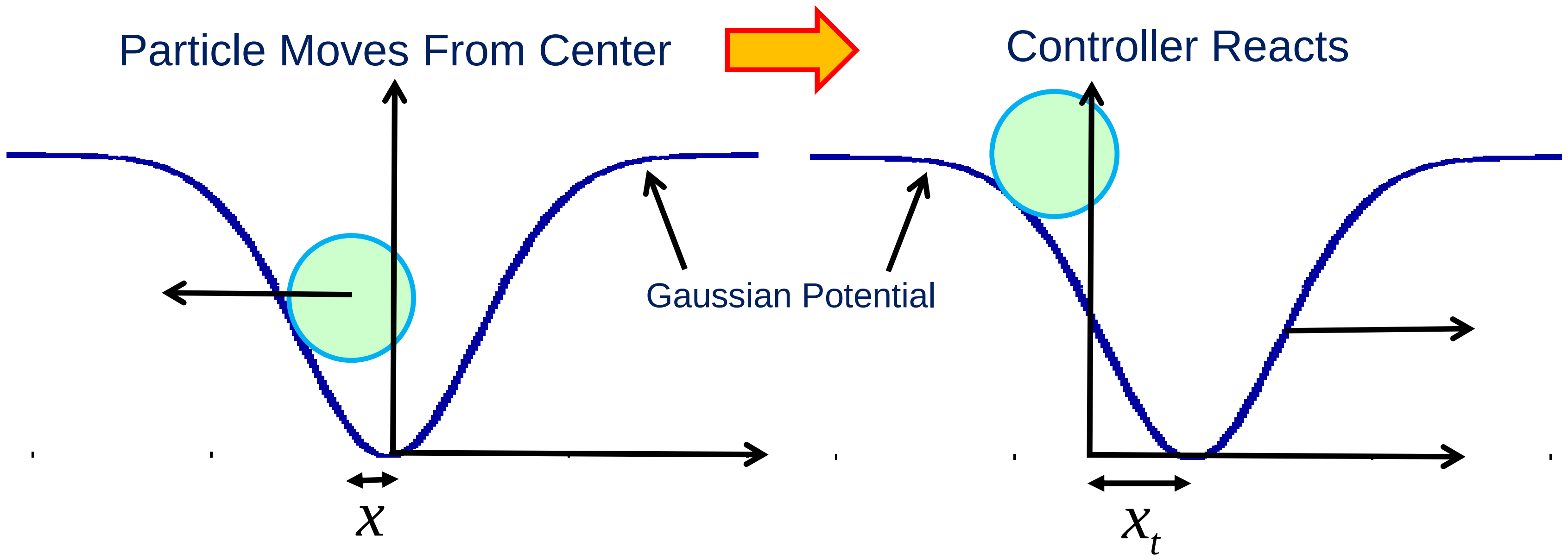
**Displacement Power Spectrum**




- This is a significant oversimplification
- However, it provides a straightforward design approach that can be understood by OT scientists and *it works*

# Scanning Control Concept

- In order to suppress Brownian motion, the trap must move in the opposite direction of the particle.
- This risks moving the particle to the top of the potential where it may escape from the trap.
- The controller must be designed to maximize Brownian motion suppression while minimizing $\bar{x} = x - x_t$
- This is achieved using $H_2$ and $H_\infty$ norms of the system dynamics.

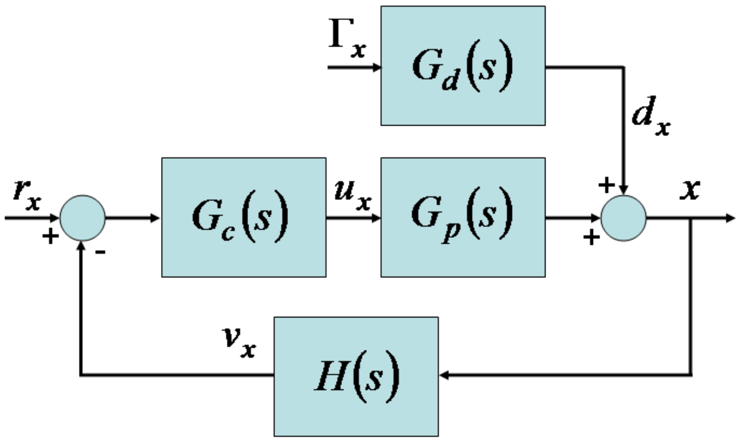


# Control System

A PID controller is selected:
•because it provides a baseline for performance
- it can easily be implemented at high bandwidth
- Scientists understand its operation

$$G_{c_x}(s) = \frac{K_d\, s^2 + K_p\, s + K_i}{s}$$

Control Block Diagram




## Closed-Loop Dynamics

$$x(s) = G_{rx}(s) r_x(s) + G_{\Gamma_x x}(s) \Gamma_x(s)$$

where:

$$G_{\Gamma_x x}(s) = \frac{bs}{(1+\bar{a}\, K_d)\ s^2 + (a + \bar{a}\, K_p)\ s + \bar{a}\, K_i}$$

This controller was implemented:
•In simulation using Matlab/Simulink
- Experimentally using an all-analog PID controller with 100 kHz bandwidth

# Control System Optimization

The control gains, $K_p$, $K_i$, and $K_d$, were designed to:
•Minimize the $H_2$ and $H_\infty$ norms for $\left|G_{\Gamma_x x}\right|$ this will reduce the Brownian motion
•Minimize the $H_\infty$ norm for $\left|G_{\Gamma_x \bar{x}}\right|$ to minimize the chance of the particle leaving the trap
•These norms can be calculated analytically since the system is low order

$$\left\|G_{\Gamma_x x}\right\|_\infty = \frac{b}{a + \bar{a} K_p}$$

Only $K_p$ can reduce the max. displacement

$$\left\|G_{\Gamma_x x}\right\|_2 = \frac{b}{\sqrt{2\left(a + \bar{a} K_p\right)\left(1 + \bar{a} K_d\right)}}$$

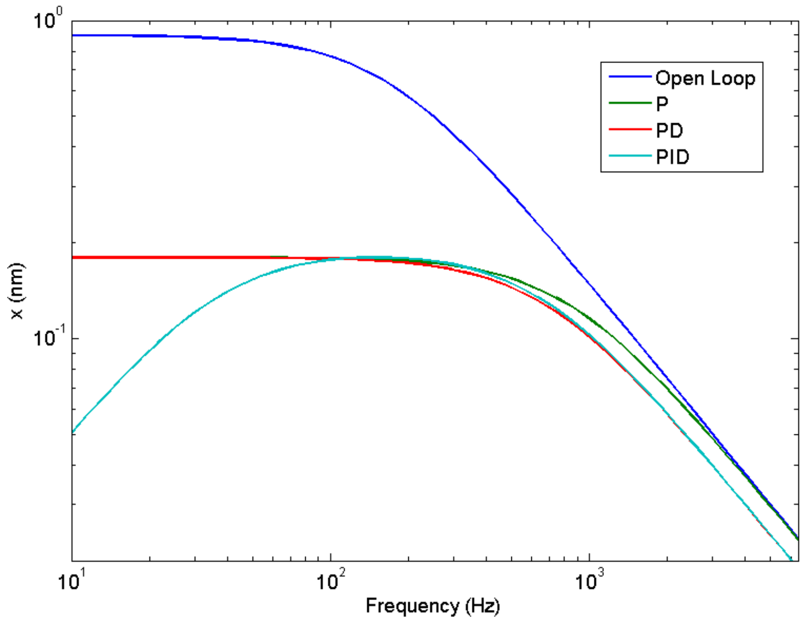
Only $K_p$ and $K_d$ can reduce the RMS displacement

$$\left\|G_{\Gamma_x \bar{x}}\right\|_\infty = \frac{b}{a}\sqrt{1 + \frac{2}{\frac{\left(a + \bar{a} K_p\right)^2}{\bar{a} K_i} - 1 + \sqrt{1 + \frac{\left(a + \bar{a} K_p\right)^2}{\bar{a} K_i}\left(1 + 2\bar{a} K_d\right)}}}$$

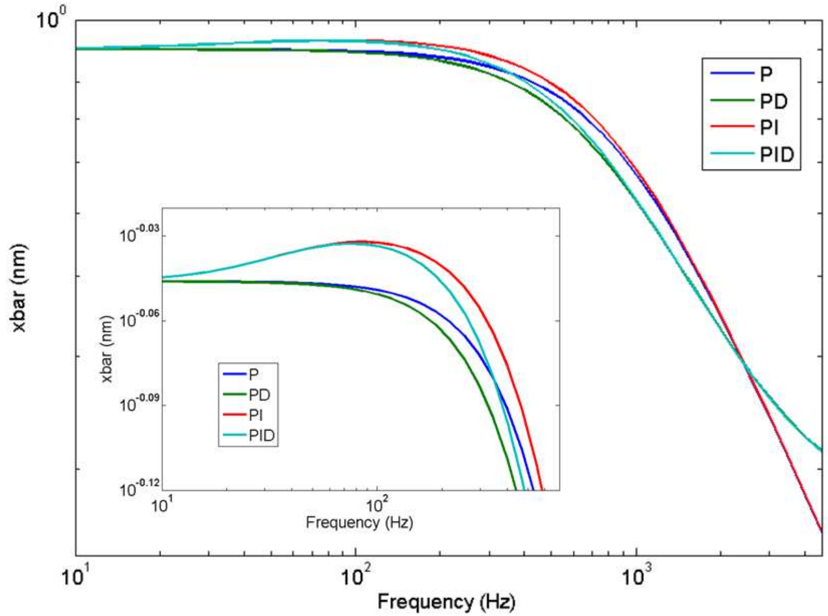
$K_i$ is only useful to shift the max. peak and attenuate Brownian motion at low frequencies

# Simulation Results

$\left|G_{\Gamma_x x}\right|$ without a time delay



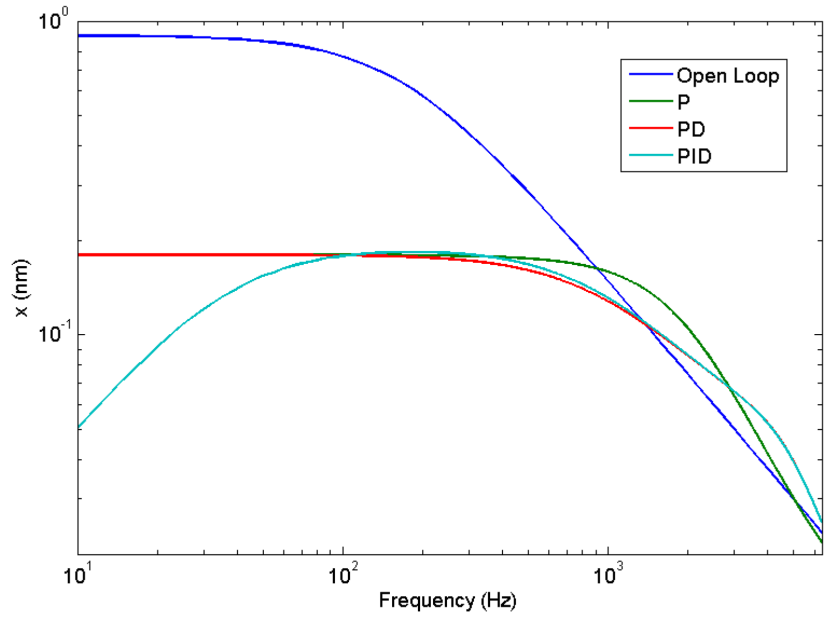

$\left|G_{\Gamma_x \bar{x}}\right|$



# Simulation Results

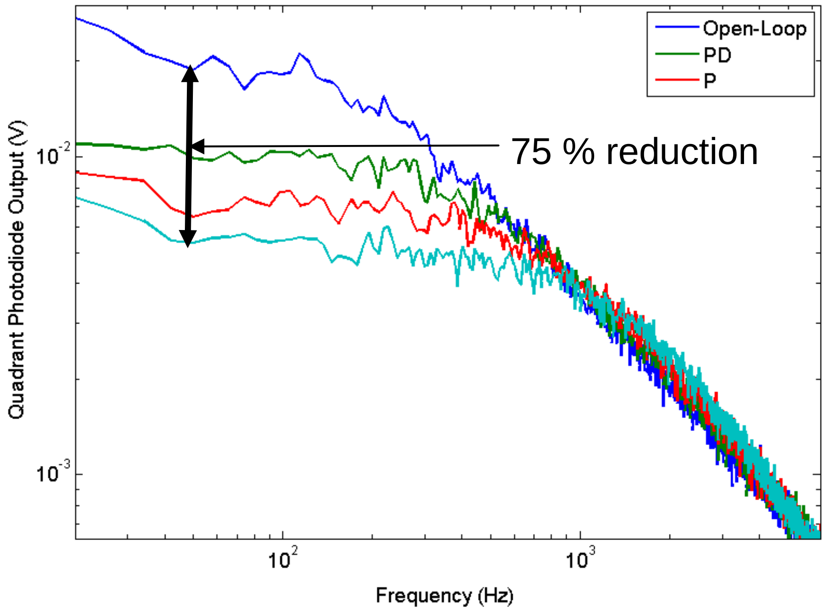
| Controller | $\lVert G_{\Gamma_x x} \rVert_\infty$ (nm) | $\lVert G_{\Gamma_x x} \rVert_2$ (nm) |
|---|---|---|
| Open Loop | 0.897 | 20.50 |
| P | 0.180 | 9.198 |
| PD | 0.180 | 8.279 |
| PID | 0.180 | 8.279 |

$\left| G_{\Gamma_x x} \right|$ with a time delay

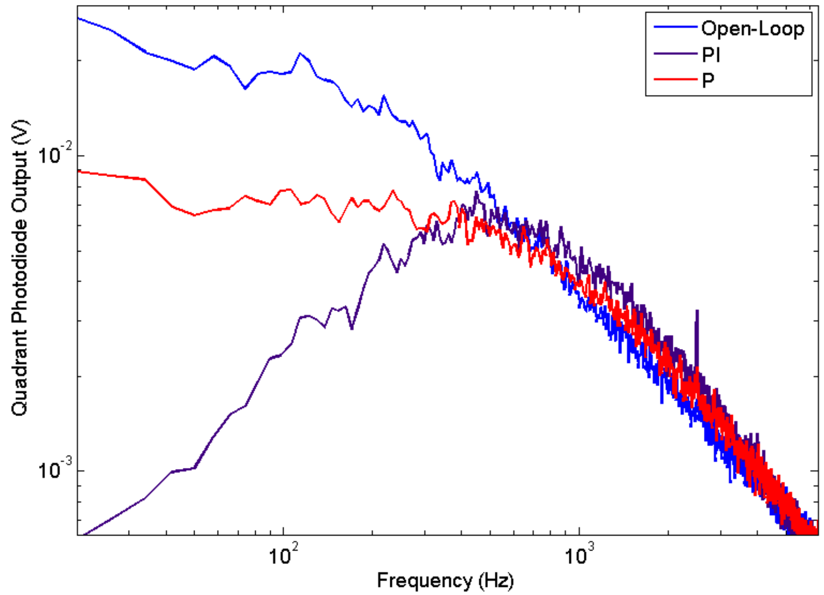


# Experimental Results

## Proportional (P) Control

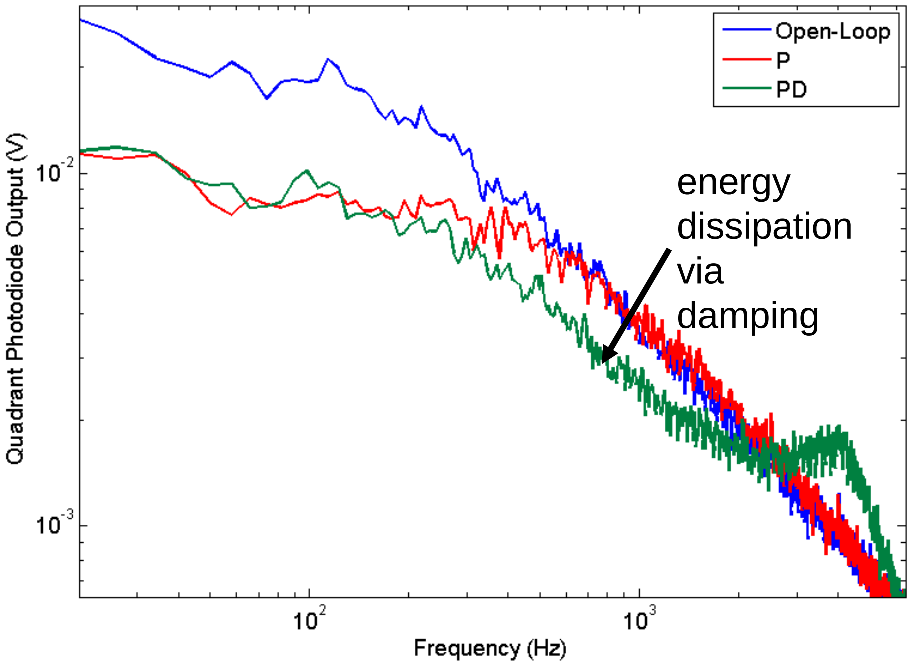



## Proportional – Integral (PI) Control



# Experimental Results

## Proportional-Derivative (PD) Control




| Case | Gains | $\lVert G_{\Gamma_x x} \rVert_\infty$ | $\lVert G_{\Gamma_x x} \rVert_2$ |
|---|---|---|---|
| 1 | Open Loop | 0.0282 | 0.0395 |
| 2 | $K_p = 7.6$ | 0.0109 | 0.0286 |
| 3 | $K_p = 10.0$ | 0.0115 | 0.0267 |
| 4 | $K_p = 13.6$ | 0.0090 | 0.0248 |
| 5 | $K_p = 19.6$ | 0.0077 | 0.0220 |
| 6 | $K_p = 10.0, K_d = 3.5e^{-4}$ | 0.0119 | 0.0232 |
| 7 | $K_p = 19.6, K_i = 2000$ | 0.0077 | 0.0242 |

# Conclusions

- The current approach results in:
  - 75 % reduction in max. displacement
  - 45 % reduction in RMS displacement
- The derivative term appears to “cool” the particle
- Although functional, the time delay in the AOD is limiting performance. VCOs will solve this problem.
- Integral control is only useful for shifting the noise into a higher frequency range. This has benefits.
- The next step is to tackle this problem with nonlinear control.
- Surprisingly, there are gaps in nonlinear stochastic control theory that make this a challenging problem

# Future Work

- Brownian motion suppression using intensity control via the EOM
- Integration of intensity and scanning control for 3 DOF suppression
- Apply these techniques to increase the trapping lifetime of nanoparticles below 100 nm